Astralon: A Mathematical Exploration of Constellation-Like Structures

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Abstract

Astralon is a newly proposed field in mathematics that investigates the properties and dynamics of constellation-like structures within various mathematical frameworks. This paper rigorously develops the foundations of Astralon, introduces new notations, and derives relevant formulas.

1 Notation and Definitions

1.1 Constellation Graph

A *constellation graph* $\mathcal{C}$ is defined as a finite simple graph $G = (V, E)$ where V represents stars (vertices) and E represents the connections (edges) between them.

- **Vertices** (V): Represent the stars in a constellation.
- **Edges** (E): Represent the connections between stars, forming recognizable patterns.

1.2 Constellation Map

A *constellation map* $\mathcal{M}$ is a function $\mathcal{M} : \mathcal{C} \to \mathbb{R}^2$ that maps the constellation graph $\mathcal{C}$ onto a 2D plane, representing the celestial sphere.

- **Coordinate Map**: Each vertex $v_i \in V$ is assigned coordinates $(x_i, y_i) \in \mathbb{R}^2$.

1.3 Star Distance and Angle

The distance between two stars v_i and v_j is given by the Euclidean distance:

$$d(v_i, v_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

The angle between three stars v_i, v_j, v_k forming a triangle is given by:

$$\theta_{ijk} = \arccos\left(\frac{(v_j - v_i) \cdot (v_k - v_j)}{\|v_j - v_i\|\|v_k - v_j\|}\right)$$

2 Properties of Constellation Graphs

2.1 Constellation Connectivity

A constellation graph $\mathcal{C}$ is said to be connected if there exists a path between any two vertices.

2.2 Constellation Degree

The degree $\deg(v_i)$ of a vertex $v_i \in V$ is the number of edges incident to v_i.

2.3 Constellation Subgraphs

A subgraph $\mathcal{C}' = (V', E')$ of $\mathcal{C}$ is a graph where $V' \subseteq V$ and $E' \subseteq E$.

3 Dynamics of Constellation Graphs

3.1 Constellation Transformation

A *constellation transformation* is a mapping $T : \mathcal{M} \to \mathcal{M}'$ that alters the positions of stars while preserving the overall pattern.

- **Translation**: $T_{\text{trans}}(v_i) = (x_i + a, y_i + b)$
- **Rotation**: $T_{\text{rot}}(v_i) = (x_i \cos\phi - y_i \sin\phi, x_i \sin\phi + y_i \cos\phi)$
- **Scaling**: $T_{\text{scale}}(v_i) = (\lambda x_i, \lambda y_i)$

3.2 Constellation Stability

The stability of a constellation graph under transformation is defined by its invariant properties:

$$\mathcal{C} \equiv \mathcal{C}' \iff \forall v_i, v_j \in V, d(v_i, v_j) = d(T(v_i), T(v_j))$$

4 Astralon Formulas and Theorems

4.1 Constellation Area

The area of a constellation $\mathcal{C}$ can be computed by triangulating the graph and summing the areas of the constituent triangles. For a triangle $\triangle v_i v_j v_k$:

$$A_{ijk} = \frac{1}{2} \left| x_i(y_j - y_k) + x_j(y_k - y_i) + x_k(y_i - y_j) \right|$$

The total area A of the constellation is:

$$A = \sum_{\triangle v_i v_j v_k \subseteq \mathcal{C}} A_{ijk}$$

4.2 Constellation Center of Mass

The center of mass (x_c, y_c) of the constellation $\mathcal{C}$ is given by:

$$x_c = \frac{1}{|V|} \sum_{v_i \in V} x_i, \quad y_c = \frac{1}{|V|} \sum_{v_i \in V} y_i$$

5 Advanced Topics in Astralon

5.1 Constellation Homotopy

Two constellation graphs $\mathcal{C}$ and $\mathcal{C}'$ are homotopic if there exists a continuous transformation $H : \mathcal{C} \times [0, 1] \to \mathcal{M}$ such that:

$$H(\mathcal{C}, 0) = \mathcal{M}, \quad H(\mathcal{C}, 1) = \mathcal{M}'$$

5.2 Constellation Spectral Analysis

The spectrum of a constellation graph $\mathcal{C}$ is the set of eigenvalues of its adjacency matrix A.

$$A_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \in E \\ 0 & \text{otherwise} \end{cases}$$

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of A. The spectral radius ρ of $\mathcal{C}$ is defined as:

$$\rho = \max_{1 \leq i \leq n} |\lambda_i|$$

6 Conclusion

Astralon offers a rich mathematical framework for studying constellation-like structures through the lens of graph theory, geometry, and topology. By introducing new notations and formulas, we have laid the foundation for rigorous exploration and further development in this exciting new field.

References

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